UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 11

Option gamma. The delta-gamma-theta approximation.

Please, provide your **complete solution** to the following problem. Final answers without shown reasoning will get zero points.

Problem 11.1. (5 points) Assume the Black-Scholes model. Let the current price of a non-dividend-paying stock be equal to \$80 per share. Its volatility is 0.20.

The continuously compounded risk-free interest rate is 0.04.

Consider a one-year, at-the-money European call option on the above stock. The current delta of the call option is 0.6179. What is the current gamma of the call option?

Solution: Using our IFM tables, or differentiating, we know that

$$\Gamma_C(S(0),0) = \frac{e^{-\delta T} N'(d_1(S(0),0))}{S(0)\sigma\sqrt{T}} = \frac{N'(d_1(S(0),0))}{S(0)\sigma}$$

From the given value of the call's delta, we have that

$$N(d_1(S(0),0)) = 0.6179 \quad \Rightarrow \quad d_1(S(0),0) = 0.3.$$

So, the gamma of the call equals

$$\Gamma_C(S(0),0) = \frac{N'(0.3)}{80(0.2)} = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{0.09}{2}}}{16} = 0.0238367.$$

Problem 11.2. (5 points) Assume the Black-Scholes model. Let the current stock price be \$100. Consider an option on this stock such that its current price is \$3.65, its delta is -0.4182, and its gamma is 0.016. What will the approximate price of this option be should the stock price rise to \$104 in a small time interval?

Solution: We use the delta-gamma approximation, and get

$$\begin{aligned} v(S(dt),dt) &\approx v(S(0),0) + \Delta(S(0),0)(ds) + \frac{1}{2}\Gamma(S(0),0)(ds)^2 \\ &= 3.65 + (-0.4182)(4) + \frac{1}{2}(0.016)(4)^2 = 2.1052. \end{aligned}$$

Problem 11.3. (5 points) Assume the Black-Scholes model. Bertie Wooster was looking at stock-price and option data from yesterday. He decides to pose his friend Tuppy Glossop a riddle. Bertie tells Tuppy the following about yesterday's price of a stock and information on an option on this stock:

- the stock price yesterday was greater than \$77;
- the option's price was \$2.45;
- the option's delta was -0.1814;
- the option's gamma was 0.04;
- the option's theta was 0.01 **per day**.

Tuppy is allowed to see today's stock price and today's option price. They turn out to be \$80 and \$2.20, respectively. What is Tuppy going to guess to be yesterday's stock price?

Solution: Using the delta-gamma-theta approximation, we get that

$$2.20 = 2.45 + (-0.1814)ds + \frac{1}{2}(0.04)(ds)^2 + 0.01.$$

Simplifying the above quadratic equation, we obtain

$$0.02(ds)^2 - 0.1814ds + 0.26 = 0 \quad \Leftrightarrow \quad (ds)^2 - 9.07ds + 13 = 0.$$

Solving for ds, we get

$$ds_{1,2} = \frac{9.07 \pm \sqrt{9.07^2 - 4(13)}}{2} = \frac{9.07 \pm 5.50135}{2}.$$

The two solutions are

$$ds_1 = 1.7843 \quad \text{and} \quad ds_2 = 7.2857.$$

We conclude that yesterday's stock price was \$78.22.